# Compass and Straightedge Constructions

MTH 202

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**Construction 1 - Equilateral triangle given a side**

Given a segment $\overline{AB}$, an equilateral triangle with sidelengths $AB$ can be constructed by:

(1) Draw a circle with center $A$ and radius $AB$.

(2) Draw a circle with center $B$ and radius $AB$.

(3) Mark an intersection point between the two circles, denoted $C$.

Then $\Delta ABC$ is an equilateral triangle, because all the sides have length $AB$.

**Construction 2 - Bisect a segment $\overline{AB}$**

Given a segment $\overline{AB}$, we can find the midpoint (or line passing through the midpoint perpendicular to $\overline{AB}$) by:

(1) Draw a circle with center $A$ and radius $AB$.

(2) Draw a circle with center $B$ and radius $AB$.

(3) Mark both intersection points between the two circles, denoted $C$ and $D$.

(4) Draw the segment $\overline{CD}$.

The intersection of $\overline{CD}$ and $\overline{AB}$ is the midpoint of $\overline{AB}$, and the line $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.

**Construction 3 - Bisect an angle $\angle UVW$**
Given a vertex $V$ and rays $\overrightarrow{VU}$ and $\overrightarrow{VW}$, we can bisect angle $\angle UVW$ by:

(1) Draw a circle with center $V$.

(2) Mark the intersection points with rays $\overrightarrow{VU}$ and $\overrightarrow{VW}$ as $A$ and $B$.

(3) Find the midpoint of $\overline{AB}$ using **Construction 2**. Denote it as $C$.

(4) Draw ray $\overrightarrow{VC}$.

The ray $\overrightarrow{VC}$ bisects the angle $\angle UVW$.

**Construction 4 - Draw a line perpendicular to line $L$ through point $P$ on $L$**

(1) The line forms a straight angle at $P$. Bisect this angle using **Construction 3**.

(2) Extend the ray constructed in Step 1 to a line.

This line is perpendicular to $L$ and passes through $P$.

**Construction 5 - Draw a line perpendicular to line $L$ through point $P$ not on $L$** These two constructions overlap heavily:

(1) Draw a circle with center $P$ intersecting line $L$.

(2) Mark the intersection points as $A$ and $B$.

(3) Find the midpoint of segment $\overline{AB}$ using **Construction 2**, denoted as $C$.

(4) Construct line $\overleftrightarrow{PC}$.

The line $\overleftrightarrow{PC}$ is perpendicular to $L$.

**Construction 6 - Copy triangle $\Delta ABC$ onto a ray $\overrightarrow{DE}$**
Denote the start of the ray as $A'$ (as well as $D$). Then:

(1) Draw a circle with center $A'$ and radius $AB$.

(2) Mark the intersection point with $\overrightarrow{DE}$ as $B'$.

(3) Draw a circle with center $A'$ and radius $AC$.

(4) Draw a circle with center $B'$ and radius $BC$.

(5) Mark the intersection point between these points as $C'$.

Then $\Delta A'B'C'$ is a copy of $\Delta ABC$ since all the sidelengths are the same in both triangles.

The following constructions are also useful, but we won't cover them explicitly in class:

**Construction 7 - Copy an angle onto a ray**
This is a slight modification of the previous. Start with $\angle UVW$ and a ray $\overrightarrow{RS}$:

(1) Mark a point $B$ on $\overrightarrow{VU}$.

(2) Mark a point $C$ on $\overrightarrow{VW}$.

(3) Copy $\Delta BUC$ onto ray $\overrightarrow{RS}$ using **Construction 6**.

Angle $\angle R$ is congruent to $\angle UVW$.

**Construction 8 - Line parallel to $L$ through point $P$**
This constructs two perpendicular lines:

(1) Construct the line $M$ through $P$ perpendicular to $L$ using **Construction 5**.

(2) Construction the line $N$ through $P$ perpendicular to $M$ using **Construction 4**.

Line $N$ is parallel to $L$.